INAUGURAL LECTURE

THE QUANTUM HALL EFFECT

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Education:

Professor Shrivastava obtained his B,Sc. degree from the University of Agra in first class in 1961, M.Sc. degree from the University of Allahabad in first class in 1963, Ph. D. degree from the Indian Institute of Technology in 1966 and D.Sc. degree from the Calcutta University in 1980.

Fellowships:
He was elected fellow of the Institute of Physics, U.K., in 1985 and also Charted Physicist from U.K. He was elected fellow of the National Academy of Sciences, India, fellow of the Ultrasonic Society of India, fellow of the Indian Cryogenics Council, etc. He is a member of the American Physical Society.

Appointments:
He was invited to work in USA many times. During 1966-68, he worked at the Clark University, Worcester, Massachusetts, during 1968-69 at the Montana State University. Here partial salary support was provided by NASA. During 1974-75 he worked in the University of California in Santa Barbara and during 1987 in the University of Houston, Texas.

For one year, 1969-70, he worked in the University of Nottingham in the U.K., for one year, 1975-76, in the State University, Utrecht, in the Netherlands, for one year, 1986-87, in the Concordia University in Montreal, in Canada and in 1999-2000, in the Tohoku University, Sendai, in Japan.

He has worked in the Harvard University in USA, in the Royal Institute of Technology, Stockholm and in the Uppsala University in Sweden. He has held visiting appointments in the University of Zurich in Switzerland and in the International Centre for Theoretical Physics in Italy.

He was Professor in the University of Hyderabad from 1978-2005.
He was appointed Professor in the University of Malaya in December 2005.

Publications: **180 papers. 4 books.**
Production**. He has produced 18 Ph.D., 25 M.Phil., 12 M.Tech. and many M.Sc. theses.**
Areas of research:
He has worked in electron spin resonance, relaxation, spin-phonon interaction, Mossbauer effect, fractals, magnons, superconductivity, semiconductors, charge-density waves, spin-polarized hydrogen, quantum Hall effect, density functional theory, Racah algebra, Green functions, etc.

Abstract

The Hall effect is the generation of a current perpendicular to the direction of applied electric as well as applied magnetic field in a metal or a semiconductor. It is used to determine the concentration of electrons. The Quantum Hall effect has been discovered by von Klitzing in Germany and by Tsui, Stormer and Gossard in U.S.A. Robert Laughlin also in U.S.A. explained the quantization of Hall current by using "flux quantization" and introduced incompressibility to obtain fractional charge. We have developed the theory of the quantum Hall effect by using the theory of angular momentum. Our predicted fractions are in accord with those measured. Von Klitzing in 1985, and Tsui, Stormer and Laughlin in 1998 received the Nobel prize for their discoveries. In this lecture, we report a review of the subject as well as emphasize our explanation of the observed phenomena. We use spin to explain the fractional charge and hence we discover spin-charge coupling and compare our explanation with other prize winning researches.

Introduction

We have been working for the last 40 years. Our major contributions to the knowledge of Physics are in the following areas:

(i) Phonon-induced contribution to the spin-Hamiltonian.
(ii) EPR measurement of Debye temperature.
(iii) Relaxation processes.
(iv) New Shift in the Moessbauer effect.
(v) New contact charge density.
(vi) Exchange and superexchange interactions between electrons.
(vii) Magnon-photon interaction.
(viii) Soft-mode from electronic splitting in doped ferroelectrics.
(ix) Semiconductors, electron-hole recombination times.
(x) Superconductivity, magnetic superconductors
(xi) Discovery:Flux quantized imaginary part of response function.
(xii) Spin-polarized hydrogen atoms.
(xiii) Fractals.
(xiv) ^{4}He.
(xv) Quantum Hall effect. Spin dependent flux quantization, spin-charge locking.
(xvi) Density-Functional Theory: Modelling with atoms.
(xvii) Nanocrystals, specific heat of nanocrystals.
(xviii) Brillouin scattering from nanocrystals, new kink in dispersion.
(xix) Vibrations in glasses, etc.

Mathematical Tools:

Quantum Mechanics, Racah Algebra, Green Functions, Quantum Field Theory.

History.

The ordinary Hall effect was discovered by Edwin Hall in 1879. In 1930, Landau showed that the orbital motion of the electron gives magnetic susceptibility. In 1978 K. von Klitzing and Th. Englert found a plateau in the Hall effect. In 1980 von Klitzing found the value of h/e^2 from the plateau in the Hall effect. In 1985, Klitzing was awarded the Nobel prize in Physics for the discovery of quantum Hall effect. In 1982 D. C. Tsui, H. L. Stormer and A. C. Gossard discovered the steps at fractional numbers. In 1998, Tsui, Stormer and Laughlin were awarded the Nobel prize.

Applications.

(i) Hall probe. The detection of magnetic fields is often done by using Hall currents.
(ii) Hall effect sensors. The sensor responds to changes in the magnetic field intensity.
(iii) Hall effect motors and switches.

The force is $\mathbf{F} = e\,\mathbf{v}\times\mathbf{B}$ so that the Hall voltage is, $V = IB/necd$
where I is the Hall current, B is the magnetic induction, n the electron concentration, e is the electron charge, c is the velocity of light and d is the thickness of the sample as shown in the figure 1.

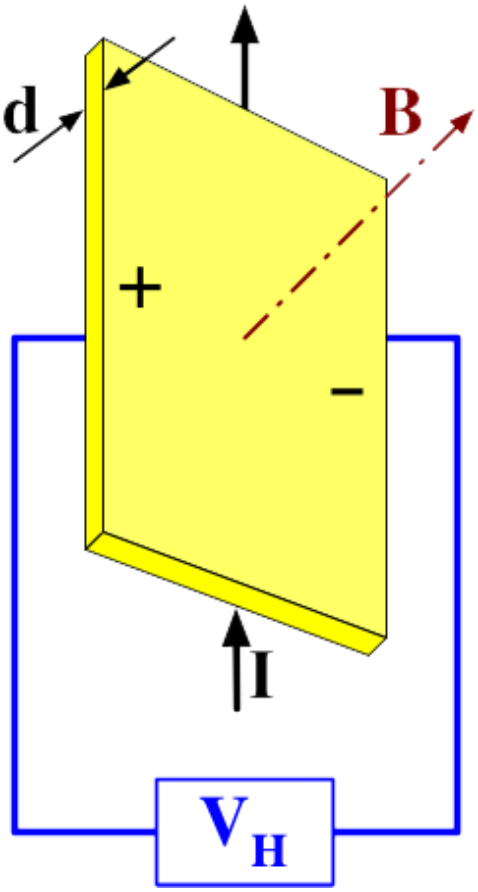


Fig.1 The Hall voltage is measured orthogonal to both the applied electric as well as the magnetic field.

Two Dimensional Electron Systems:

A two dimensional electron system is formed in a heterostructure which has layers of GaAs over AlGaAs. The energy gap of GaAs increases upon Al doping. When GaAs is doped with donors at zero temperature the Fermi level lies higher than the bottom of the conduction band. The electrons bound to donors move into GaAs conduction band and the process stops when some proportion of electrons have moved. The electrons in the inversion layer are two dimensional as shown in Fig.2.

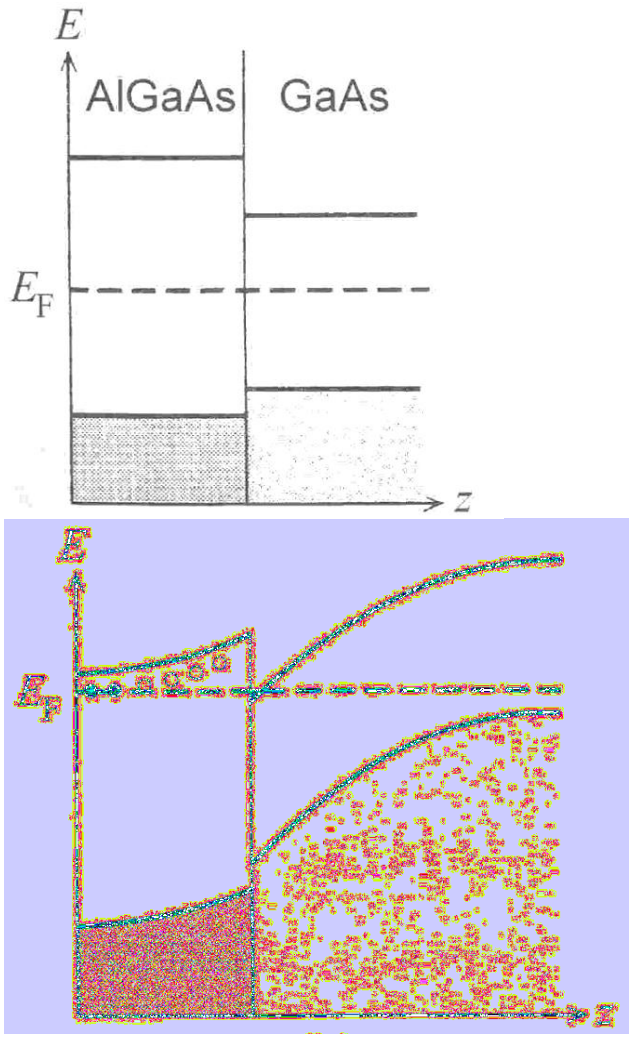


Fig.2 To make a two-dimensional electron system, layers of AlGaAs are deposited over GaAs.

The average drift velocity of the electron subjected to the electric field is,

$V_d = -eE\tau/m$ (1)

where E is the electric field, m is the electron mass and τ is the mean life time so that the current density is,

$j = -neV_d=\sigma_oE$ (2)

where

$\sigma_o= ne^2\tau/m$ (3)

where n is the electron density. In the presence of a steady magnetic field, the conductivity and resistivity become tensors/matrices,

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \tag{4}$$

We take x and y axes in the 2-dimensional plane, to obtain,

$i_x = \sigma_{xx} E_x + \sigma_{xy} E_y$,

$i_y = \sigma_{yy} E_x + \sigma_{yx} E_y$. (5)

Owing to the isotropy, $\sigma_{xx}=\sigma_{yy}$ and $\sigma_{xy}=-\sigma_{yx}$. The first one is called the diagonal conductivity and the second one is called the Hall conductivity. The relation between conductivity and resistivity is,

$$\rho_{xx}=\rho_{yy}= \frac{\sigma_{xx}}{\sigma_{xx}^{\ 2} + \sigma_{xy}^{\ 2}} \tag{6}$$

for the diagonal resistivity and for the Hall resistivity, we obtain,

$$\rho_{xy}= - \rho_{yx}= - \frac{\sigma_{xy}}{\sigma_{xx}^{\ 2} + \sigma_{xy}^{\ 2}} \tag{7}$$

We measure these quantities by connecting various leads as given in Fig. 3.

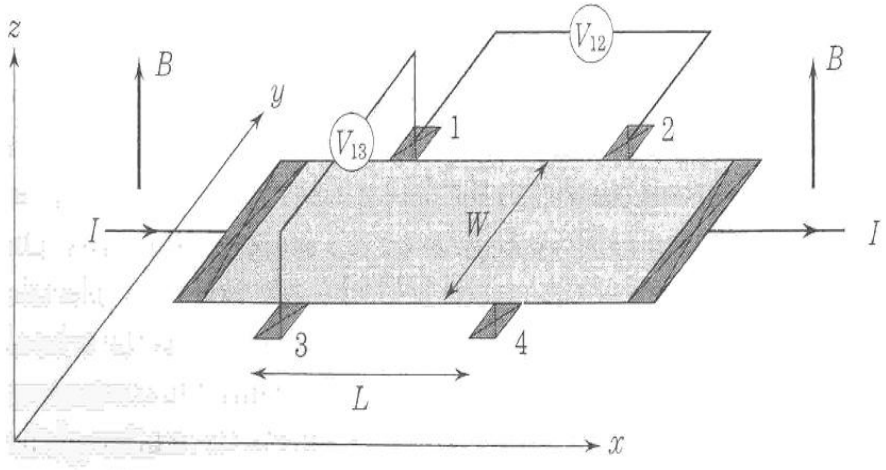


Fig.3. The sample with the magnetic field B and the current I perpendicular to it. The wires are connected to measure Hall voltage perpendicular to both I and B in various directions. W is the width of the sample and L is the length.

In the case of homogeneous current in the y direction, $i_x=I/W$ and $i_y=0$. The electric field is given by, $E_x=V_{12}/L$ and $E_y=V_{13}/W$ so we have, $\rho_{xx}=V_{12}W/IL$ and $\rho_{yx}= V_{13}/I=R_H$. The Hall resistivity in two dimensional electron system is the Hall resistance per unit area. According to the Drude (ordinary non-interacting metals) theory,
$\rho_{xx}=1/\sigma_o=m/ne^2\tau$ and $\sigma_{xy}=B/nec$ in a weak magnetic field. The Hall resistivity is inversely proportional to the electron density and independent of mean scattering life time. In strong magnetic fields, there are new phenomena. Von Klitzing found the plateau

in the Hall resistivity which gave the correct value of h/e^2. One of his plateaus is given in Fig.4. We see from the plot that (i) there is a plateau region in which the Hall resistivity remains constant. As the electron density is varied in this region the diagonal resistivity

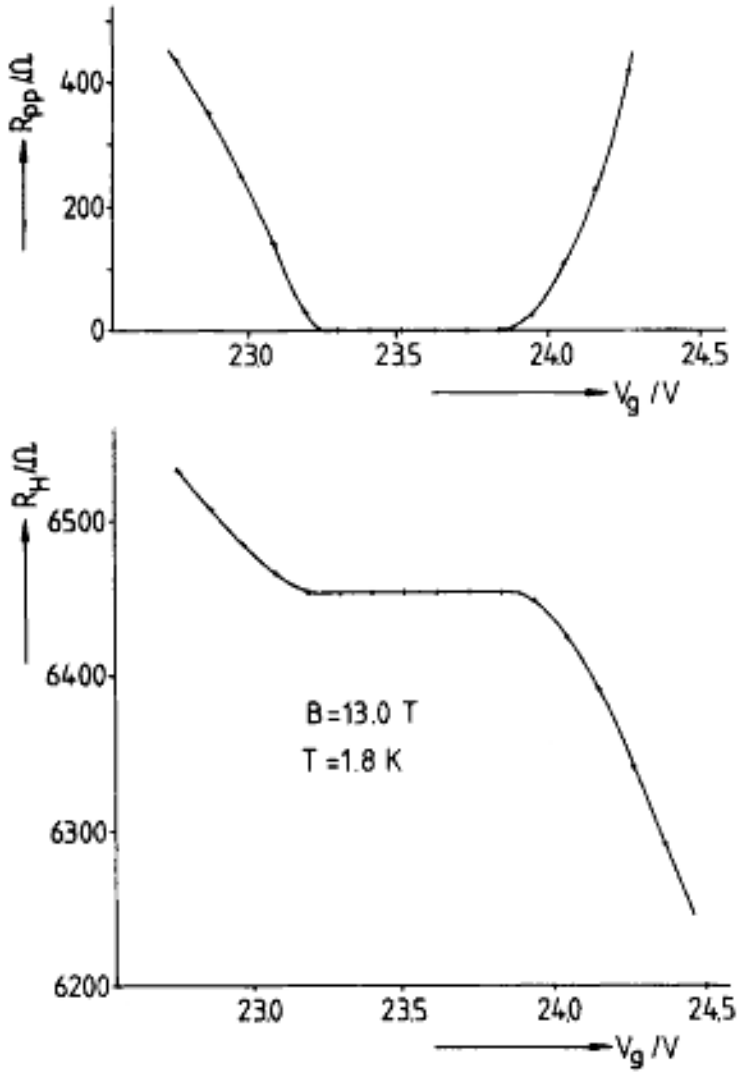


Fig.4. The Hall resistivity shows plateau at high fields. The picture shows von Klitzing's data at a field of 13 Tesla ($13x10^4$Gauss).

is almost zero. (ii) The value of the Hall resistivity in the plateau regions is exactly equal to h/e^2 divided by an integer. Therefore, the Hall conductivity σ_{xy} in the plateau region is "quantized" into integer multiples of e^2/h. This phenomenon was called the "integer quantized Hall effect (IQHE).

Flux Quantization and the Hall effect.

Introduction of the flux quantization immediately explains von Klitzing's plateau. We have the Hall resistivity as,

$\rho = B/nec$ (8)

where B is the magnetic field. According to flux quantization, the field in a certain area, A, is quantized,

$B.A=m\varphi_o$ (9)

where the magnetic flux quantum is, $\varphi_o=hc/e$. Substitution of (9) into (8) gives the integer quantized Hall effect as,

$$\rho = \frac{\frac{mhc}{Ae}}{\frac{N_o}{A}ec} = \frac{m}{N_o}\frac{h}{e^2} \quad . \qquad (10)$$

The quantum Hall effect thus is the quantization of Hall resistivity as,

$$\rho = \frac{h}{ie^2} . \qquad (11)$$

Hence the charge of the quasiparticle is ie. Here i = integer. The charge thus becomes 1e, 2e, 3e, …, ie, … . von Klitzing obtained the correct value of the charge for i=1. So the value of $h/1e^2$ became "one von Klitzing". When this experiment was repeated with cleaner samples with higher electron mobility, with higher magnetic fields and lower temperatures, it lead to the discovery of i =1/3 plateau which gave birth to the fractional charge. We show the data of Tsui, Stormer and Gossard in Fig. 5 with plateau at 1/3. Since this fractional value occurred in the middle of various highly degenerate Landau levels, where no gap is apparent, the observation could not be explained by the experimentalists by using the non-interacting quantum mechanical theory. It was thought that the observation is a result of the many-body effects of electron interactions. Subsequent experimental work showed a lot many more plateaus which are displayed in Fig.6. We will see that there is no need of interaction to explain the plateau at 1/3.

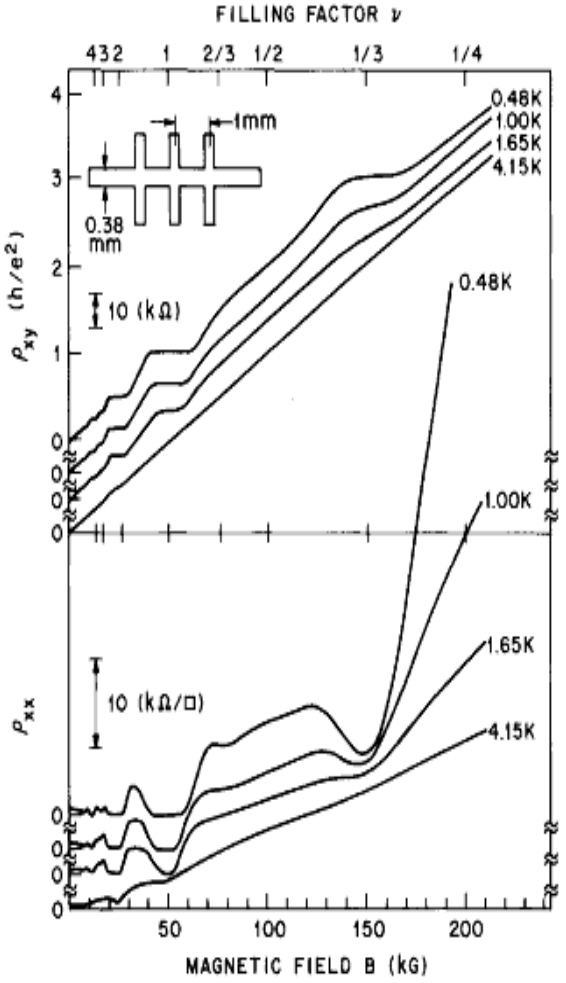


Fig.5. The data of Hall resistivity showing plateau at 1/3 below a temperature of 1 K.{D.C. Tsui, H.L.Stormer and A.C. Gossard).

Laughlin's Theory:

Laughlin made the first efforts to explain the quantum Hall effect. The flux quantization was immediately found to explain at least the integer quantized Hall effect. Subsequently, Laughlin started from first principles using the Hamiltonian,

$$H = \sum_j \{\left|(\hbar / i)\nabla_j - (e/c)\vec{A}_j\right|^2 + V(z_j)\} + \sum_{j>k} e^2 / \left|z_j - z_k\right| \qquad (12)$$

where j and k sum over N particles and V is the potential due to nuclei. The repulsive Coulomb interactions can produce the plateaus only when flux quantization is considered. A trial wave function of the form given below,

$$\left|\Psi(z_1,...,z_N)\right|^2 = e^{-\beta(\phi+\gamma)} \qquad (13)$$

with $\beta=1/m$,

$$\phi = -2m^2 \sum_{j<k} \ell n \left| z_j - z_k \right| + m/2 \sum_{\ell}^{N} \left| z_\ell \right|^2 \qquad (14)$$

and γ is a multicenter integral, to solve the Schrodinger equation to find the ground state.

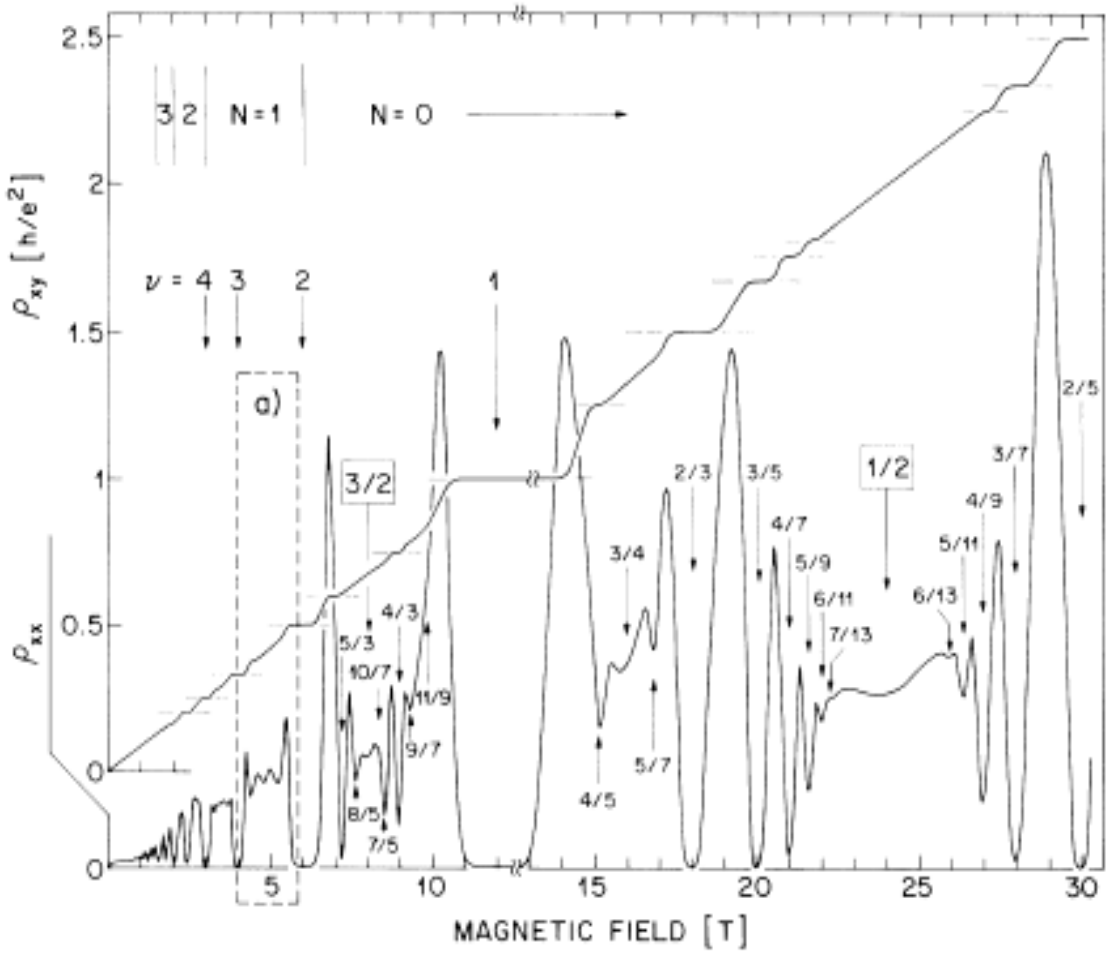


Fig.6. The fractional charges seen symmetrically around ½.

Laughlin obtained the approximate energy expression in terms of m as well as computed exactly. By using "incompressibility" the charge of the particles is fixed at 1/3 and 1/5. Unfortunately, there is area in the flux quantization which must also be fixed, otherwise the charge will leak. Laughlin also shared the 1998 Nobel prize with Tsui and Stormer.

Shrivastava's Theory.

We consider that electrons have spin as well as the orbital angular momentum so that,

$$g_j j = g_s \vec{s} + g_l \vec{l} = \frac{1}{2}(g_l + g_s)\vec{j} + \frac{1}{2}(g_l - g_s)(\vec{l} - \vec{s}) \quad (15)$$

Multiplying both sides by j = l +s and taking eigen values,

$$g_j j(j+1) = \quad (16)$$

$$1/2(g_l + g_s)\, j(j+1) + 1/2(g_l - g_s)[(\vec{l}\,(l+1) - s(s+1)]$$

Substituting s=1/2 and j = $l \pm (1/2)$ we get,

$$g_j = g_l \pm \frac{g_s - g_l}{2l+1} \quad (17)$$

For g_s=2, g_l = 1, we find,

$$g_{\pm} = 1 \pm \frac{1}{2l+1} \ . \quad (18)$$

The equation (17) has both the signs for the spin. The cyclotron frequency is,

$$\omega = \frac{eB}{mc} \quad (19)$$

From the charge, e in the cyclotron frequency, we generate the charge of a particle. It is also possible to obtain the charge from the e in Bohr magneton. Corresponding to the cyclotron frequency, the voltage along y direction is,

$$\hbar\omega = eV_y. \quad (20)$$

or,

$\hbar \frac{eB}{mc} = eV_y$. Multiplying this expression by e/h, we get,

$$\frac{e^2 B}{2\pi mc} = \frac{e^2}{h} V_y \quad (21)$$

which is the current in x direction, so that the resistivity,

$$\rho_{xy} = \frac{h}{e^2} \quad (22)$$

The S_z=1/2, and the energy in a magnetic field is $g\mu_B$H.S, so correcting B in the cyclotron frequency we find,

$$I_x = \frac{1}{2} g \frac{e^2 B}{2\pi mc} = \frac{1}{2} g \frac{e^2 V_y}{h} \quad (23)$$

For l =0, g=2,

$$I_x = \frac{e^2}{h} V_y \quad (24)$$

which describes the quantized current correctly for ν=1. From the above equations we have $\nu = \frac{1}{2} g_{\pm}$ which gives the filling factor, one for + sign and the other for – sign as in (18). For l =0, we obtain $(1/2)g_{+}=1$ and $(1/2)g_{-}=0$ and other values as given in Table 1.

Table 1. Predicted values of the fractional charge.

l	$(1/2)g_{-}=\frac{l}{2l+1}$	$(1/2)g_{+}=\frac{l+1}{2l+1}$
0	0	1
1	1/3	2/3
2	2/5	3/5
3	3/7	4/7
4	4/9	5/9
5	5/11	6/11
6	6/13	7/13
∞	1/2	1/2

The Landau levels are introduced by multiplying the above values by n so that $\nu=n(\frac{1}{2} g_{\pm})$ so we can multiply the tabulated values by an integer when needed. The values of ρ_{xx} and ρ_{xy} for a single interface of GaAs/AlGaAs have been measured by Willet et al at 150 mK. The values predicted in the table 1 are exactly the same as in Willet's experimental data shown in Fig.7. The values shown in the figure occur in two sets,
The values 2/5, 3/7, 4/9, 5/11, 6/13, … etc. and 2/3, 3/5, 4/7, 5/9, 6/11 etc. The predicted values in the table are the same as in the experimental data. Using the table 1, when we multiply the values by n we can interpret all of the experimentally measured values. The columns of table 1 belong to two Kramers conjugates, one belonging to +1/2 and the other one to -1/2 spin and the experimental data also displays them in two sets.

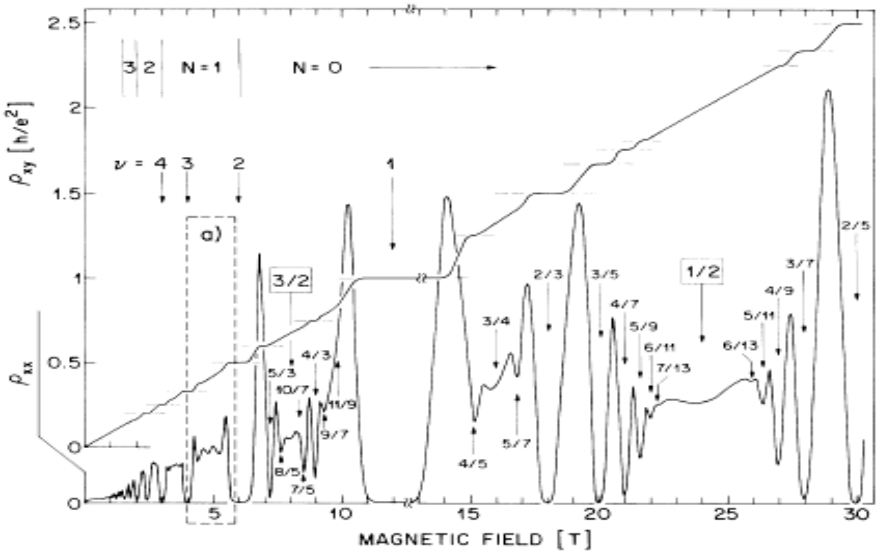
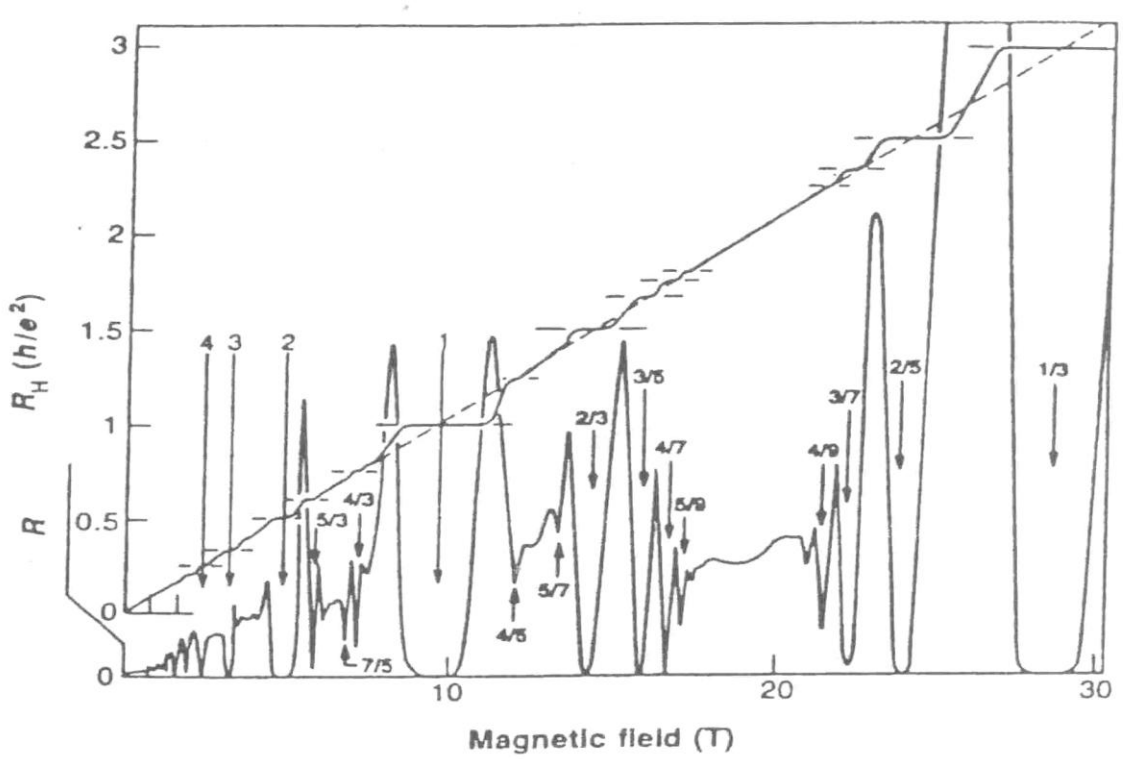


Fig.7 The experimental data on Hall resistivity showing fractions of charges.



Fiog.8. Hall effect data showing plateau at 1/3. The observed series of charges are exactly the same as in Table 1.

For the cyclotron frequency $\hbar\omega_c = g\mu_B B$ where $\mu_B = e\hbar/2mc$ is the Bohr magneton. Therefore, $(1/2)g_{\pm}$ can be considered to be the effective charge, $e_{eff} = \frac{1}{2} ge = \nu e$. In table 1, we see two series,

$\nu_- = \frac{l}{2l+1}$ and $\nu_+ = \frac{l+1}{2l+1}$ which can be used to explain the high Landau levels easily.

Eisenstein et al have found that for the higher values of the Landau level quantum number, n, the number of fractions observed are much less than at the lowest Landau level. At the magnetic field of 4 or 5 Tesla only a small number of fractions are observed, the strongest ones are at: 8/3, 5/2 and 7/3. The series $l/(2l+1)$ is the particle-hole conjugate of $(l+1)/(2l+1)$. For l=7, two values, 7/15 and 8/15 are predicted and for l=∞ the value is ½. When the same particle occurs in different levels its charge remains unchanged. We can multiply the values by n=5 so that the predicted values of ½, 7/15 and 8/15 become 5/2, 7/3 and 8/3. These predicted values are exactly the same as those observed experimentally by Eisenstein et al. Thus, 7/3 is the particle-hole conjugate of 8/3 as seen in Table 2 for n=5.

l	$l/(2l+1)$	$(l+1)/(2l+1)$	n $l/(2l+1)$	n$(l+1)/(2l+1)$
∞	1/2	1/2	5/2	5/2
7	7/15	8/15	7/3	8/3

For a long time only odd denominators were reported which show that even denominators are weak. After that even denominators as well as even numerators with odd denominators are found. We go back to the same formula, $\frac{l+\frac{1}{2}\pm s}{2l+1}$. When s=1, l = 0, the denominator becomes an even number.

The predicted fraction for l=0 is 3/2 and for l=1, it is 5/2. Hence for electron clusters or pairs the fraction has even denominator. Since the number of particles is larger than one its probability becomes small so these plateaus are weak but the same theory explains the even denominators. There is a limiting value of the series which also gives ½. One can introduce a Fermi surface at n/2, Thus ½, 2/2, 3/2, 4/2, 5/2, 6/2 and 7/2 are predicted which are observed by Yeh et al as shown in Fig.9.

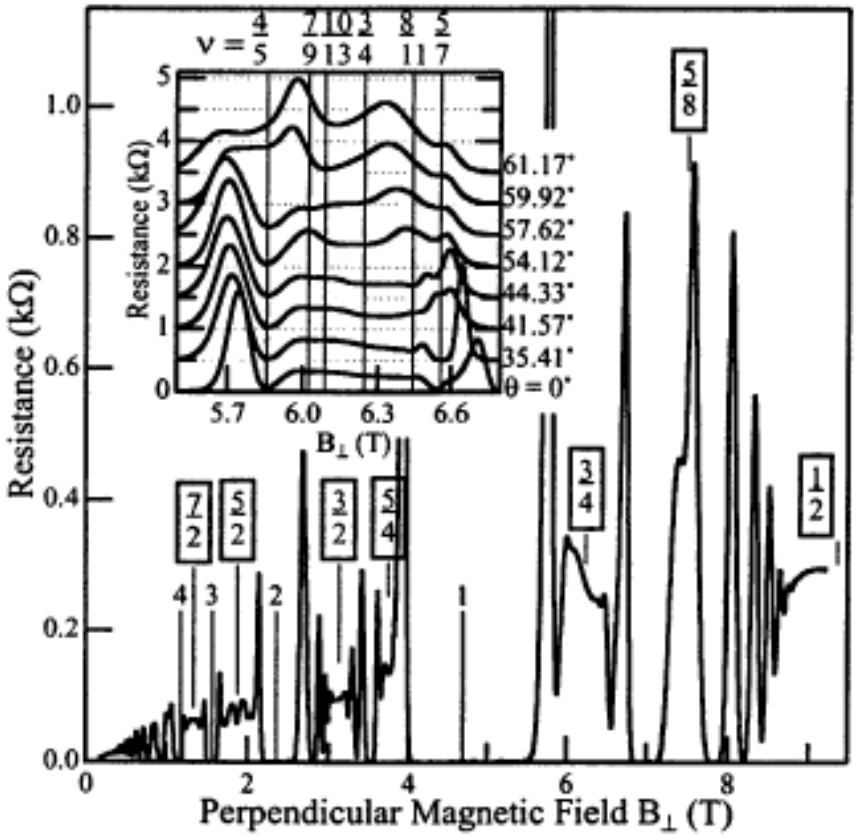


Fig.9. The predicted n/2 is observed in this experimental data.

It has been reported by Yeh et al that the effective mass and g factor of some of the fractions are equal to those of others. We have shown that the effective mass can be equal only when the two quasiparticles are particle-hole conjugates. The particle-hole conjugates should obey the following relation,

$\nu_p+\nu_h=1$ (25)

The values given in Table 1 always obey this relation.

Half-filled Landau level:

The $l = \infty$ in the two series produces,

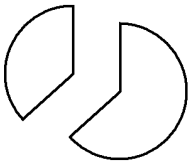
$$\lim_{l\to\infty} \nu_+ = \frac{1}{2}(+)$$
$$\lim_{l\to\infty} \nu_- = \frac{1}{2}(-) \qquad (26)$$

one ½ comes from the right and the other from the left when magnetic field is varied, i.e., one while increasing the field and the other while reducing. One can go from 1/3 to ½ by reducing the field while from 2/3 to ½ is obtained by increasing the field. We can go from 1/3 to 3/5 by reversing the spin and increasing the *l*. Similarly from 2/3 to 2/5 by reducing the spin and increasing *l*. In this way angular momentum is conserved. One of the ½ values is like an electron (1/2)A and the other is like a hole, (1/2)B, (A for + series and B for – series). Since the electron and the hole are separated by a distance, this state is compressible. When we multiply this result by n, the Landau level quantum number, we obtain: ½,2/2,3/2,4/2,5/2, …, which are in agreement with data.

Effective charge:

We discussed the Laughlin's wave function earlier. Here the repulsive Coulomb interactions can not give rise to a fractional charge of 1/3. It is first assumed on the basis of experimental data and then substituted in the theory. Laughlin's charge is independent of spin but in our theory it depends. In Laughlin's theory a particle of charge 1/3 is produced but in our theory, splitting occurs in fractions of 1/3 and 2/3, etc.

Laughlin's 1/3 charge.



Shrivastava's theory. (1/3)e+(2/3)e =1e
(2/5)e+(3/5)e=1e, etc.

Fig.10. There is a difference between Laughlin and our theories. The fractionalization in Laughlin's theory is independent of spin. Whereas in our theory spin plays an important role in determining the fraction.

Dirac equation:

The basic idea of Dirac equation is to have space-time symmetry and the constancy of velocity of light, so that instead of $p^2/2m$, the kinetic energy appears as $c\boldsymbol{\alpha}.\mathbf{p}$ and the wave equation becomes,

$$(c\alpha.p+\beta mc^2)\Psi(x,t)=i\hbar\frac{\partial}{\partial t}\Psi(x,t) \qquad (27)$$

The free particle solutions of which are,

$$E_{\pm}=\pm(c^2p^2+m^2c^4)^{1/2}. \qquad (28)$$

This equation gives the correct magnetic moments for the proton as well as for the neutron subject to using the mass of the respective particle and an appropriate g value. In the case of electron Lande's formula,

$$g=1+\frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)} \qquad (29)$$

with positive spin is used. In our case, the effective charge and hence the magnetic moment is determined by the g values. Hence, our method of defining the charge is the same as that for the magnetic moments of proton and neutron.

Shubnikov-de Haas effect:

At low temperatures, the integration over the Fermi distributon leads to x/sinh x
Type expression which is called Dingle's formula. The spin symmetry is found to modify this formula which determines the oscillation amplitude of resistivity as a function of magnetic field, called the Shubnikov-de Haas effect. Our theory introduces the effective charge so that the cyclotron frequency gets fractionalized resulting into $m/\nu_{\pm}$ which for $\nu_{\pm}=1$ becdomes m, the electron mass. Thus, we have taken into account, the spin-charge fractions, to obtain the correct mass. For example, at certain magnetic field 1.5 m is found instead of m. The mass of the electron relative to band value as a function of carrier density deduced from Shubnikov-de Haas effect in GaAs/AlGaAs heterostructures is shown in the Fig.11. The factors 1 and 2/3 are found to arise from the spin-charge effect of Shrivastava. The experimental data is obtained from Tan, Stormer et al. The dashed line is found by Kwon et al on the basis of small self energy corrections due to many body perturbative interactions. The factors of 1 and 2/3 in the mass are found by us. The mass of the free electron is m_e and the screening radius is deduced from the density, n, per unit area. We find that $m/\nu_{\pm}$ occurs in place of m for the mass of the electron in the Shubnikov-de Haas (SdH) effect. In fact, many other fractions of the mass of the electron given in Table 1, become allowed so that the electron really "falls apart". The oscillations due to flux quantization allow the measurement of m/h^2. The flux quantization in the Shubnikov-de Haas effect leads to "quantized Shubnikov-de Haas effect". Therefore, we observe consequences of the effect of flux quantization on the Shubnikov-de Haas

oscillations. There are zeroes in the resistivity at certain fields. There is a spin-charge effect so that the spin flip corresponds to a change in the charge.

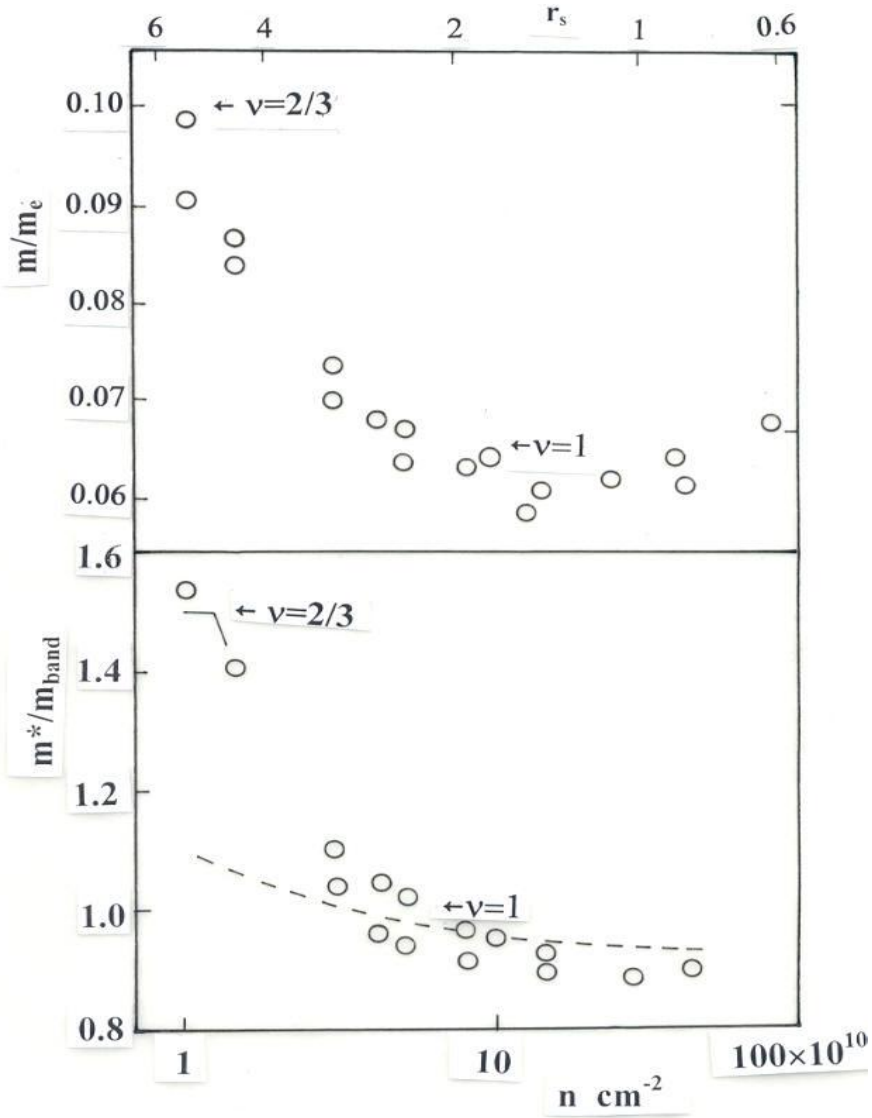


Fig.11. The mass of the electron deduced from the amplitude of oscillations as affected by our charge factors.

The Shubnikov-de Haas effect uses quantization of Landau levels but not the flux quantization. Hence, we find that there is a "quantized Shubnikov-de Haas effect" which measures the m/h^2. We find that when fractional values of $\nu_{\pm}$ are taken into account, the mass of the electron, equal to band mass in GaAs/AlGaAs is obtained. When magnetic field is varied, the different values of n cross the Fermi energy at different fields resulting into oscillations in the resistivity as a function of magnetic field. The oscillating resistivity is given by,

$$\delta\rho_{xx}(\varepsilon)_{\pm} = \rho_o \sum_p \gamma_{th} c_{\alpha,\beta} \exp\left(-\frac{p\pi}{\nu_{\pm}\omega_c\tau_q}\right)\cos\left[2\pi p\left(\frac{\varepsilon}{\nu_{\pm}\hbar\omega_c}-\frac{1}{2}\right)\right]$$

where (30)

$$\gamma_{th} = \frac{2\pi^2 p k_B T/(\hbar\omega_c\nu_{\pm})}{\sinh\left[2\pi^2 p k_B T/(\hbar\omega_c\nu_{\pm})\right]}$$

The cosine factor also leads to zero resistivity when ever,

$$2\pi p\left(\frac{\varepsilon}{\nu_{\pm}\hbar\omega_c}-\frac{1}{2}\right)=\frac{\pi}{2} \quad (31)$$

so that the resistivity vanishes when B satisfies the above formula. We introduce the flux quantization so that the exponential factor in (30) becomes,

$$\exp\left(-\frac{P\pi\ mc}{\nu_{\pm}\tau_c eB}\right)=\exp\left(-\frac{\pi\ mA}{\nu_{\pm}\tau_c h}\right) \quad (32)$$

so that m/h will be measured from the oscillations. Introducing the flux quantization in the argument of Sinh factor, we obtain,

$$\hbar\nu_{\pm}\omega_c = \frac{\hbar\nu_{\pm}n_2 h}{mA} \quad (33)$$

which does not have the charge but measures m/h. In the experiments the factor measured is m*g*/n so it is clear that the mass and g get mixed.

Spin-Charge Locking:

The charge of the electron may be described by matrices just as the angular momentum is. When the spin is aligned along the charge, such as s_x parallel to e_x, the arrangement is called the spin-charge locking. Taking our effective charge expression for $e_{eff}/e=(1/2)g_{\pm}$ we find the dot product of spin and charge to find,

$$e^*.s' = (2l+1)^{-1}\left(l+\frac{1}{2}\pm s\right).s' = (2l+1)^{-1}\left(1.s'+\frac{1}{2}(1).s'+s.s\right)$$

which produces spin-orbit and spin-spin interactions and there is spin divided by $2l+1$. That is what makes it difficult to detect this type of effect.

So what is our discovery?

We have found the correct explanation of the experimentally observed quantum Hall effect. We find that angular momentum gives rise to fractional charge. Therefore, there is a spin-charge effect, i.e., under high magnetic fields, the spin determines the charge.

THE NOBEL PRIZES WHY THEY WERE AWARDED?

1930: Sir Chandrasekhar Venkat Raman. Light scattering. In addition to the incident light, there are new frequencies which are equal to the vibrational frequencies in the sample.
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2006: John C. Mather and George F. Smoot. Galaxies and stars.
2007: Peter A. Gruenberg and Albert Fert, Magnetoresistance, lower resistivity at higher field phase.

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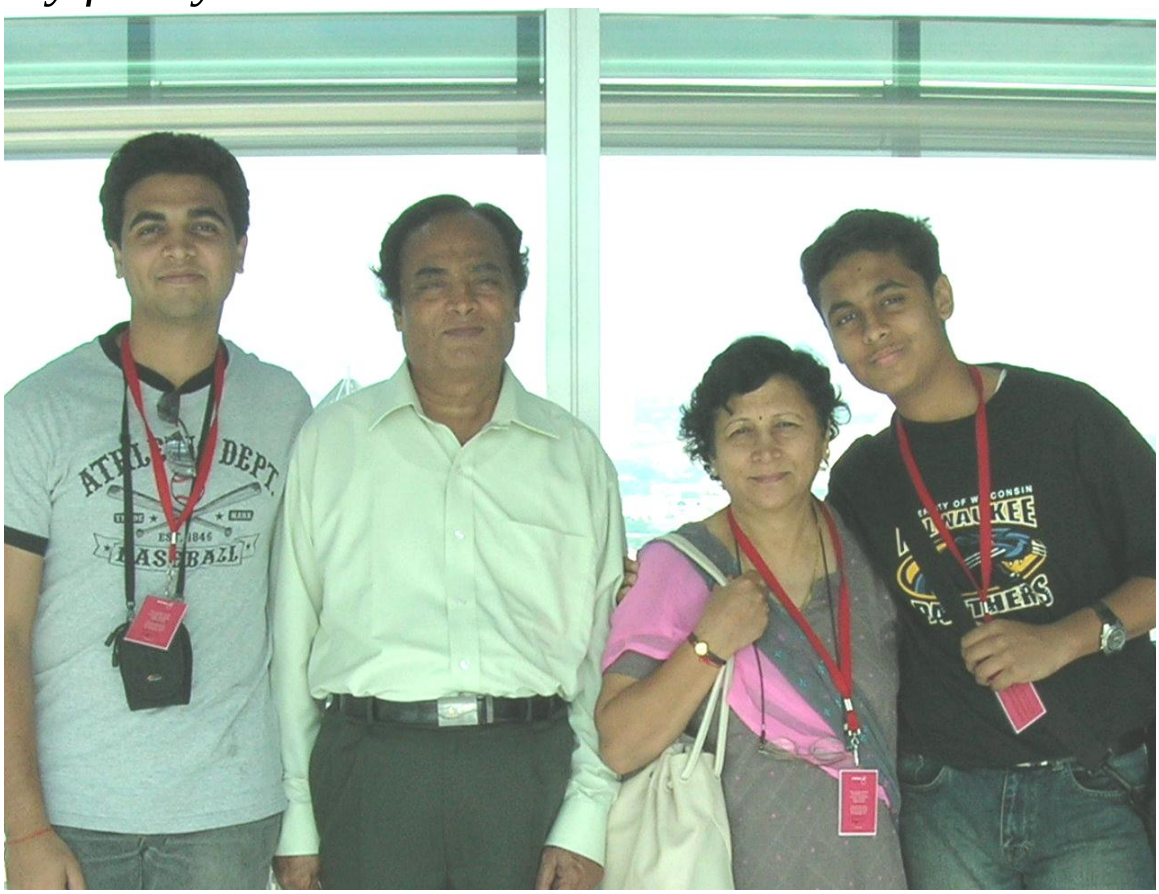

Further reading: *Page No.*

1. *K. N. Shrivastava,* Rational numbers of the fractionally quantized new states and their observation in fractionally quantized Hall effect, *Phys. Lett. A 113, 435(1986).115, 459(1986)(E).* *23*
2. *K. N. Shrivastava,* Negative-spin quasiparticles in quantum Hall effect, *Phys. Lett. A326, 469-472(2004).* *25*
3. *K. N. Shrivastava,* Particle-hole symmetry in quantum Hall effect, *Mod. Phys. Lett. B 13, 1087-1090(1999).* *29*
4. *K. N. Shrivastava,* Theory of quantum Hall effect and high Landau levels, *Mod. Phys. Lett. B 14, 1009-1013(2000).* *33*
5. *K. N. Shrivastava,* The fractional charges in the quantized Hall effect, *Nat. Acad. Sci. Lett.(India) 9, 145(1986).* *38*
6. *K. N. Shrivastava,* Spin-dependent charge of a quasiparticle in clusters in quantum Hall effect, *Nat. Acad. Sci. Lett. (India) 26, 159-161(2003). 40*
7. *K. N. Shrivastava,* Unification of charge of the electron with proton and neutron through Hall effect, *Nat. Acad. Sci. Lett.(India) 27, 435-440(2004)* *43*
8. *K. N. Shrivastava,* Microwave absorption in Wigner crystals at high magnetic fields, *Nat. Acad. Sci. Lett. 28, 199- 203(2005)* *49*
9. *K. N. Shrivastava,* The mass of the electron in Shubnikov-deHaas effect:Spin- charge locking, *Nat. Acad. Sci. Lett. (India) 29, 195-200(2006).* *54*
10. *K. N. Shrivastava,*Theory of quantum Hall effect: Effective fractional charge,*Frontiers of Fundamental Physics 4 (ed.)B.G.Sidharth and M. V. Altaisky, Kluwer Academic/Plenum Pub. New York,pp. 235-249, 2001.* *60.*